

KCL at supernode

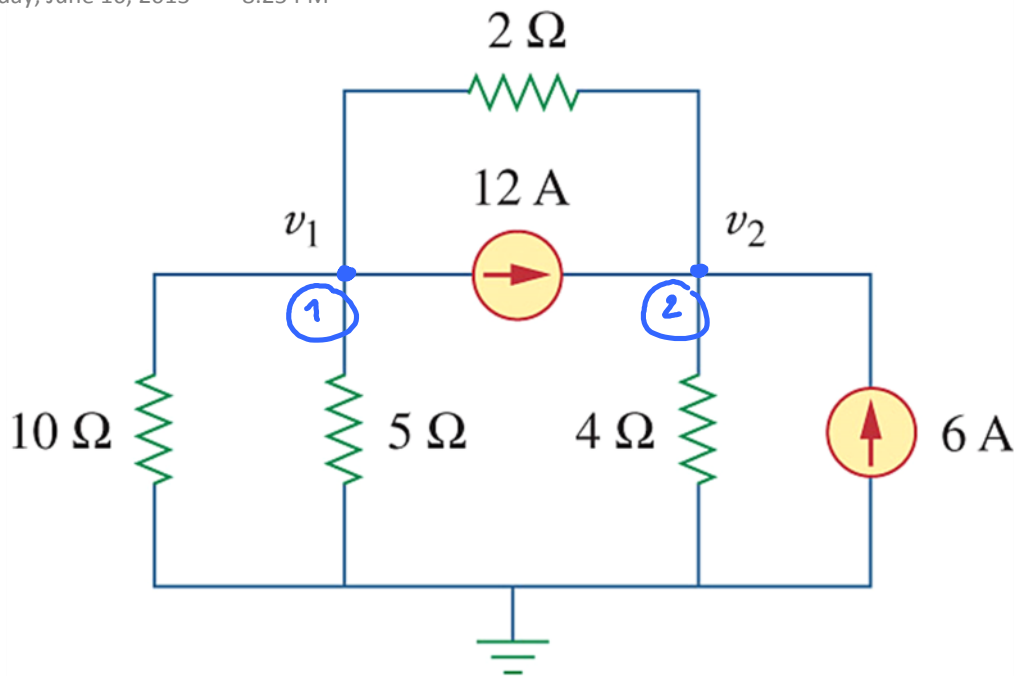
$$\frac{v_1 - 0}{4} + \frac{v_1 - v_2}{2} + \frac{v_3 - v_2}{2} + \frac{v_3 - 0}{8} = 0$$

KCL at node (2)

$$\frac{v_2 - v_1}{2} - 5 + \frac{v_2 - v_3}{2} = 0$$

Math cad

$$\begin{aligned} v_1 &= 10 \text{ V} \\ v_2 = v_3 &= 20 \text{ V} \end{aligned}$$



KCL at (1)

$$\frac{v_1 - 0}{10} + \frac{v_1 - 0}{5} + \frac{v_1 - v_2}{2} + 12 = 0$$

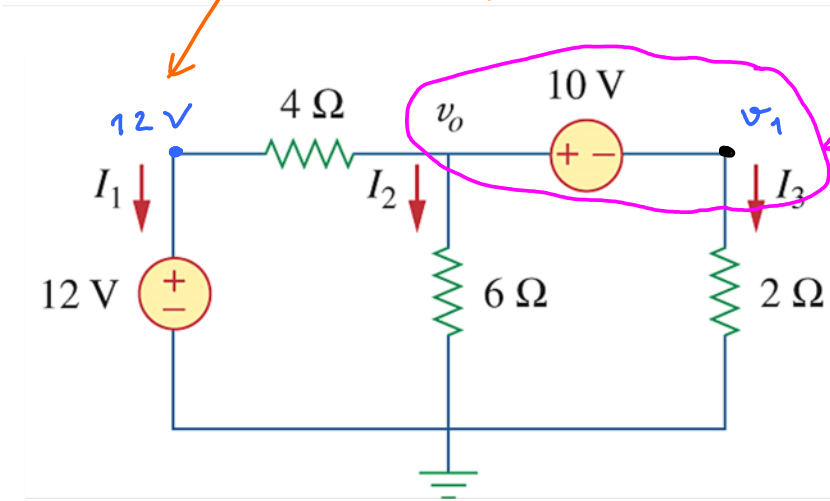
KCL at (2)

$$-12 + \frac{v_2 - v_1}{2} + \frac{v_2}{4} - 6 = 0$$

Mathcad

$$\Rightarrow \begin{cases} v_1 = 0 \\ v_2 = 24 \text{ V} \end{cases}$$

we get the voltage at this non-reference node immediately because a voltage source is between this node and the ground



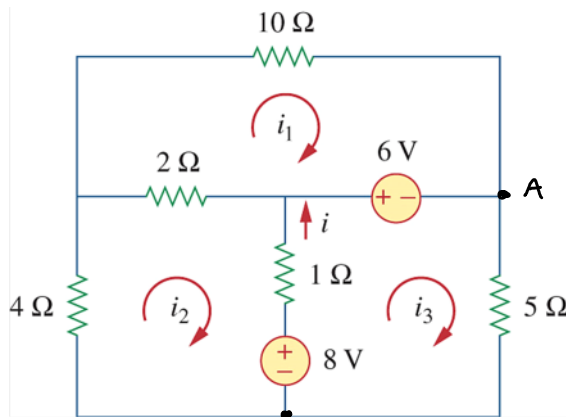
supernode
 $v_0 - v_1 = 10$

KCL at supernode:

$$\frac{v_0 - 12}{4} + \frac{v_0 - 0}{6} + \frac{v_1}{2} = 0$$

Mathcad

$$\left\{ \begin{array}{l} v_0 \approx 8.727 \text{ V} \\ v_1 \approx -1.273 \text{ V} \end{array} \right.$$



For mesh 1: We apply KVL starting from node A:

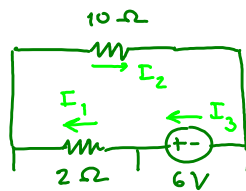
$$+6 - 2 \times (i_1 - i_2) - 10 i_1 = 0$$

Moving from - to + terminal of a 6V voltage source means I gain 6V.

Moving pass the resistor means we lose IR volts
The current here must be $i_1 - i_2$ because both i_1 and i_2 passes this 2Ω resistor.

If you feel uncomfortable with the above method for getting the equation, let's take a look at the following derivation:

In loop 1, we have



We first need to find the currents passing through these elements: I_1, I_2, I_3

It turns out that I will apply KVL later and hence I don't really care about I_3 . So, we will focus only on I_1 and I_2 .

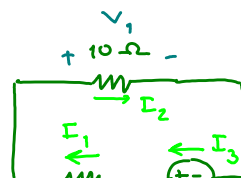
Notice how the mesh currents are defined.

We see that I_2 is the same as i_1 .

However, for I_1 we have two mesh currents i_1 and i_2 going in opposite directions. So, $I_1 = i_1 - i_2$.

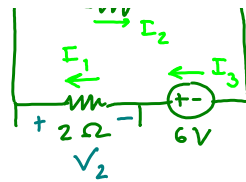
Next, we define the voltages V_1 and V_2 :

(We need the voltages because we are going to apply KVL.)



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Now, by Ohm's law, we have

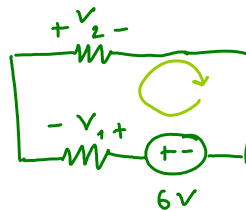


$$\begin{array}{c} V_2 \\ + \text{---} 10\Omega \text{---} \\ \text{---} \\ \rightarrow \\ I_2 \end{array} \quad V_2 = I_2 \times 10 \\ \quad \quad \quad = i_1 \times 10 \text{ V}$$

$$\text{and} \quad \begin{array}{c} I_1 \\ \leftarrow \\ \text{---} 2\Omega \text{---} \\ \text{---} \\ \downarrow \\ V_1 \end{array} \quad V_1 = I_1 \times 2 \\ \quad \quad \quad = (i_1 - i_2) \times 2 \text{ V}$$

Note that we don't have any extra minus sign in my Ohm's law because we defined the polarities of our voltages in such a way that, when considered with the directions of the branch currents, they conform with the passive sign convention.

Return to mesh 1. We now have



So, by KVL, we have

$$6 - V_1 - V_2 = 0.$$

Plugging in the Ohm's-law expressions for V_1 and V_2 , we then have

$$+6 - (i_1 - i_2) \times 2 - i_1 \times 10 = 0$$

which is the same equation that we got earlier.

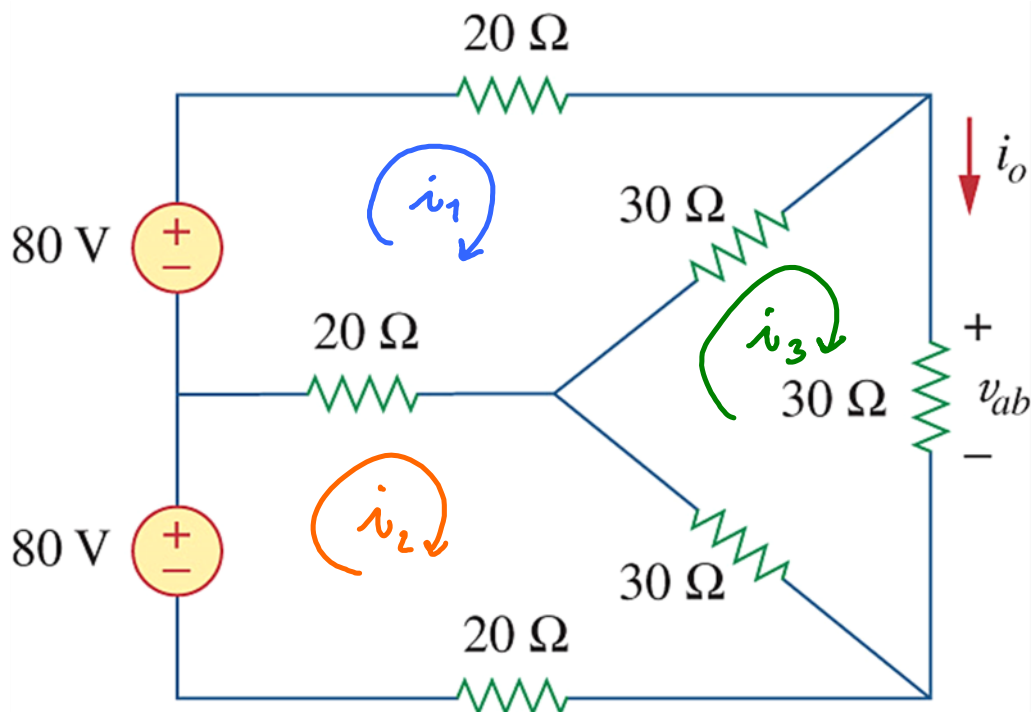
Once you apply this technique for many times, you should be able to "read" the equation directly from a loop without thinking much.

For mesh 2, starting from the bottom, we have $-4 \times i_2 - 2 \times (i_2 - i_1) - 1 \times (i_2 - i_3) - 8 = 0$

For mesh 3, starting from node A, we have $-5 i_3 + 8 - (i_3 - i_2) \times 1 - 6 = 0$

Calculator $\Rightarrow i_1 = \frac{77}{239}$, $i_2 = -\frac{40}{39}$, $i_3 = \frac{19}{117}$

Therefore, $i = i_3 - i_2 = \frac{139}{117} \approx 1.19 \text{ A}$



$$\text{Mesh 1: } 80 - i_1 \times 20 - 30 \times (i_1 - i_3) - 20(i_1 - i_2) = 0$$

$$\text{Mesh 2: } 80 - (i_2 - i_1) \times 20 - (i_2 - i_3) \times 30 - 20i_2 = 0$$

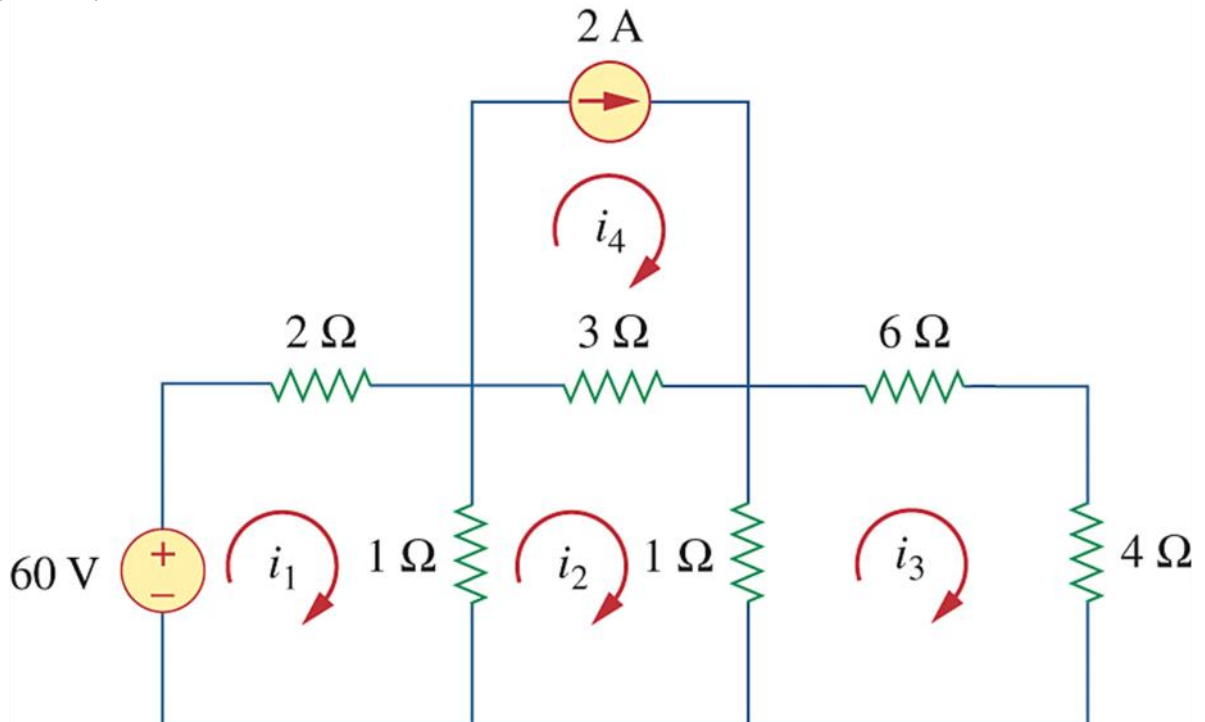
$$\text{Mesh 3: } -30(i_3 - i_2) - 30(i_3 - i_1) - 30i_3 = 0$$

$$\Rightarrow i_3 = \frac{16}{9} \quad (i_1 = i_2 = \frac{8}{3})$$

Note that $i_o = i_3$.

$$\text{So, } i_o = \frac{16}{9} \text{ A} \approx 1.78 \text{ A}$$

$$v_{ab} = i_o \times 30 = \frac{16}{9} \times 30 = \frac{160}{3} \text{ V} \approx 53.33 \text{ V}$$



In this problem, because i_4 is the only current that passes through 2A current source. We automatically get

$$i_4 = 2A.$$

$$\text{Mesh 1: } +60 - 2i_1 - (i_1 - i_2) = 0$$

$$\text{Mesh 2: } -(i_2 - i_1) - 3(i_2 - i_4) - (i_2 - i_3) = 0$$

$$\text{Mesh 3: } -(i_3 - i_2) - 6i_3 - 4i_3 = 0$$

$$\Rightarrow i_1 = \frac{3306}{151}, \quad i_2 = \frac{858}{151}, \quad i_3 = \frac{78}{151}$$

$$i_1 \approx 21.89A, \quad i_2 \approx 5.68A, \quad i_3 \approx 0.52A$$